2007 PROGRAM DIRECTOR'S CONFERENCE
SEPTEMBER 7-8 2007
WASHINGTON STATE CONVENTION AND TRADE CENTER
SEATTLE, WASHINGTON

SESSION: #977514 **TECHNICAL CONTENT ON WITHDRAWAL PORTFOLIO**

FRIDAY, SEPTEMBER 7, 2007
4:00 PM – 4:50 PM

PRESENTER:

Dr. R. Gene Stout, CFP®, Ph.D.
Central Michigan University
334 Sloan Hall
Mount Pleasant, MI 48859

Dr. Stout is a Professor of Finance and Program Director at Central Michigan University. He has more than 30 years of teaching experience and has been certified since 1999. His recent research interests employed Monte Carlo simulation to retirement portfolio and distribution planning, and he recently authored "Adjusting Fixed-horizon Retirement Planning Models for Mortality" (Journal of Business Disciplines) and "Dynamic Retirement Withdrawal Planning" (Financial Services Review).

Stochastic Optimization in Retirement Portfolio Management and Withdrawal Planning

R. Gene Stout, Professor
Department of Finance and Law
Central Michigan University
334 Sloan Hall
Mt. Pleasant, MI 48859
Ph: 989-774-6459, Fax: 989-774-6456
R.Gene.Stout@cmich.edu

Abstract

This paper applies stochastic optimization to a Monte Carlo simulation model to identify the optimal portfolio allocation for minimizing the probability of exhausting the retirement portfolio, termed portfolio ruin, from constant inflation-adjusted withdrawals subject to the uncertainties of both portfolio returns and retiree remaining lifetimes. The results indicate that the stochastically optimal portfolio allocation to equities increases by 4% for each quarter-point increase in the real withdrawal rate and 1.5% per year of earlier retirement. Optimal portfolio allocations generate probabilities of ruin that increase by about 2.35% for each quarter-point increase in the real withdrawal rate and 1% per year of earlier retirement.

Optimal results from a variable withdrawal rate model show that optimal withdrawal management from optimal portfolios reduces the average probability of ruin from unmanaged optimal portfolios by about 50% and reduces the sensitivity of both the optimal portfolio allocation and the probability of ruin to both changes in the initial withdrawal rate and the retirement age.

1. Introduction

Retirement withdrawal planning models or distribution planning models have employed Monte Carlo simulation to examine the sustainability of a constant stream of inflation-adjusted withdrawals until death, thereby incorporating the uncertainty of portfolio returns with the uncertainty of remaining retiree lifetime. More recently, Monte Carlo simulation has been used to examine the effects withdrawal management decision rules in variable withdrawal models. These models integrate decision rules which specify allowable increases and decreases in the inflation-adjusted withdrawals in response to unexpected portfolio performance. Outputs from these simulations include the distribution of the probability of running out of money before death, the distribution of portfolio values at death, and, in variable withdrawal rate models, the distribution of lifetime withdrawal rates.

Retirement withdrawal planning models typically hypothesize one or more portfolio asset allocations, since the optimal asset allocation is unknown, and similarly, variable withdrawal models also hypothesize the withdrawal management decision rules, since the optimal decision rule parameters are unknown.

This paper applies stochastic optimization to a Monte Carlo simulation model to identify the optimal portfolio allocation for minimizing the probability of retirement portfolio ruin from constant inflation-adjusted withdrawals subject to the uncertainties of both portfolio returns and retiree remaining lifetimes. Optimal portfolio allocations and probabilities of portfolio ruin are regressed on the withdrawal rate and age at retirement to quantify the impacts of earlier retirement and/or higher withdrawal rates on optimum portfolio allocations and the minimum probability of ruin.

Stochastic optimization is also applied to a Monte Carlo simulation model of variable withdrawal rates, to identify both the optimal portfolio allocations and the optimal decision-rule limitations on withdrawal rate responses to unexpected portfolio performance, for minimizing the probability of portfolio ruin. Optimal portfolio allocations and probabilities of portfolio ruin are regressed on the initial withdrawal rate and age at retirement to quantify the impacts of earlier retirement and/or higher initial withdrawal rates on optimal portfolio allocations and the minimum probability of ruin from the optimum portfolios and the optimally-managed variable withdrawal rates.

The withdrawal management process improves the performance of optimum retirement portfolios by reducing the probability of ruin, increasing the lifetime average withdrawal rates, and reducing the excess portfolio accumulation. Furthermore, the withdrawal management process reduces the sensitivities of the portfolio asset allocation and the probability of ruin to higher initial withdrawal rates and earlier retirement ages.

2. Literature Review

Previous research has explored the sustainability of inflation-adjusted withdrawals from a retirement portfolios for a fixed number of years. Studies by Bengen (1994, 1996, 1997), Tezel (2004), and Cooley, Hubbard, and Walz (1998) employ actual sequences of historic equity returns to conclude that aggressive portfolio allocations improve portfolio sustainability over long periods; and Milevsky (2001) and Ameriks, Veres, and Warshawsky (2001) use simulated market returns to support that conclusion.

Relatively fewer authors have examined the sustainability of constant inflation-adjusted withdrawals over the uncertain retirement lifespan. Terry (2003) examines portfolio sustainability over a fixed, 30-year planning period but implicitly recognizes that the risk of outliving the retirement portfolio increases with the portfolio allocation to bonds. More explicitly, Milevsky (2001) and Milevsky and Robinson (2005) integrate the investment returns and mortality risks to show that the probability of ruin is increased by either unexpectedly low investment returns or superannuation. Stout (2006) finds that the addition of mortality to a long-term fixed withdrawal rate analysis would, in general, reduce the probability of portfolio ruin by about one-half, and Stout and Mitchell (2006) support that finding with simulated results from a portfolio of 65% equities and 35% bonds.

Collins, Lawson and Chambers (1997) advise monitoring value of the retirement portfolio for withdrawal rate sustainability and reducing the withdrawal rate if the portfolio insufficiency threatens withdrawal rate sustainability. Following this advice, several authors have employed decision rules to adjust withdrawal rates as a means of preserving the retirement portfolio from ruin over a fixed term of years.

Pye (2000) employs a simulation model which permits reductions in the real withdrawal rate to improve portfolio sustainability. If poor market performance causes the current withdrawal rate to be unsustainable over the over the remainder of the fixed planning horizon at the expected portfolio rate of return, the real withdrawal rate is reduced to the withdrawal rate that *is* sustainable over the remaining planning horizon at the expected portfolio rate of return. Subsequent improvements in market performance might permit subsequent increases in the withdrawal rate, but the withdrawal rate can never exceed the initial withdrawal rate. Pye reports that market returns with an inflation-adjusted mean of 9% and standard deviation of 18% sustains a 4.5% withdrawal rate for 20 years in 85% of the portfolios without withdrawal rate reductions. The poorest performing 5% of portfolios, however, suffer withdrawal rate reductions as high as 41%, even as the median value of all portfolios nearly doubles. Pye also suggests that future analyses might permit withdrawal rates above the initial rate if the initial rate has become sufficiently sustainable. Guyton (2004) defines the safe initial portfolio withdrawal rate as the initial rate 1) which is sustainable for 40 years, 2) increases with inflation up to 6% per year, and 3) is never reduced from any previous year. He examines decision rules controlling portfolio management, withdrawals, and permissible inflationary increases in withdrawals which, collectively, increase the safe initial portfolio withdrawal rate from 4.4% to 5.8% of a 65% equities portfolio and from 4.7% to 6.2% of an 80% equities portfolio.

Following Pye's suggestion that future analyses might permit withdrawal rates above the initial rate if the initial rate has become sufficiently sustainable, Bengen (2001) reports that performance-adjusted withdrawal rates within -10% and +25% of the initial inflation-adjusted withdrawal rate, from a tax-deferred portfolio with 63% equities and

37% intermediate-term U.S. government bonds, improves the never-failing initial withdrawal rate from 4.15% to 4.58%. Guyton and Klinger (2006) also recognize the retirees' preference to maximize retirement withdrawals and to maintain purchasing power in retirement while minimizing the probability of portfolio ruin. Accordingly, they modify Guyton's earlier withdrawal rule to allow affordable increases in the withdrawals to replace prior inflationary losses, and they incorporate two new decision rules to further manage withdrawals: the capital preservation rule and the prosperity rule. They report that initial withdrawal rates of 5.2%-5.6% are sustainable at the 99% confidence level for 40 years from a portfolio of 65% equities. Furthermore, greater equity allocations and greater diversification across equities (mixed-equity classes) improves portfolio performance. Klinger (2007) reports the impacts of changing the parameters of his earlier decision rules and employs the decision-rule parameters to construct three profiles (aggressive, uniform, progressive) of withdrawal patterns over a 40-year retirement span. He also finds decision rules productive in increasing the initial withdrawal rate without increasing the probability of portfolio ruin.

Stout and Mitchell (2006) used simulation results to show that a 4.5% initial inflation-adjusted withdrawal rate from a 65% equities portfolio could be managed to increase the lifetime average withdrawal rate of a 65-year old retiree to 6.63% while reducing both the probability of ruin and excess portfolio accumulations.

Finally, McCarthy (2006) notes the growing importance of distribution planning and the importance of Monte Carlo simulations in distribution planning and communicating its trade-offs to distribution planning clients.

3. Research Design and Data

Monte Carlo simulation randomly generates values for uncertain outcomes from known or hypothesized distributions of input variables. The randomly generated outcomes are combined in a computer model to imitate real-life outcomes, usually in processes which would otherwise be too mathematically complex or too difficult to generate. Monte Carlo simulation outputs are typically statistical distributions of one or more output variables, based upon thousands of randomly generated values of input variables.

Stochastic optimization and stochastic constrained optimization are processes or algorithms to optimize a statistical property, usually the mean, of a stochastic variable, usually a simulated output variable. The Box method[1] of stochastic optimization begins by developing an initial set of input values which impact the simulated output variable. The initial set of input values will also meet a set of constraints on the output variable value, if such a set of input values exists, in the case of constrained optimization. The algorithm then iteratively searches the input values to improve the value of the simulated output, replacing the least optimal input values with more optimal values, until the statistical property of the simulated output variable converges to a single optimal solution, if such a solution exists.

---

[1] The method is described in detail in Box, M. J., 1965. A New Method of Constrained Optimization and a Comparison With Other Methods, *The Computer Journal*, Vol. 8, Issue 1, pp. 42-52

This research applies Monte Carlo stochastic optimization to two withdrawal planning models to make several important contributions to the retirement withdrawal planning literature. The first model is characterized by constant inflation-adjusted, beginning-of-the-year withdrawals from a portfolio of large-cap common stocks and intermediate-term U.S. government bonds over the uncertain retirement lifespan. Stochastic optimization is employed in this model to identify the portfolio allocation to equities which minimizes the probability of portfolio ruin. Important contributions include the application of the stochastic optimization methodology and the relationships between the optimal portfolio allocation, the constant withdrawal rate, the minimum probability of ruin, and age at retirement.

The second model is characterized by variable, but managed, inflation-adjusted withdrawals. Withdrawal rate changes are managed by decision rules patterned after Stout and Mitchell (2006). The decision rules are designed to reduce the withdrawal rate if portfolio sustainability is threatened by market under-performance or superannuation and to increase the withdrawal rate if excess portfolio accumulation or excess longevity is assured by market over-performance. Stochastic optimization techniques are used in the second model to identify both the optimal portfolio allocation and the optimal withdrawal management decision rules to minimize the probability of ruin over the uncertain remaining lifespan. Additional contributions to the withdrawal planning literature are the relationships between the model variables and the impact of withdrawal management on the probability of ruin and the optimal portfolio asset allocation.

Simulations performed as part of the stochastic optimization process use Latin Hypercube sampling[2] over 50,000 iterations[3] to identify optimum values of input variables. Subsequent simulations, employing the optimal input values to determine the value of an output variable, are performed 40 times with 50,000 iterations each.[4] Therefore, even as the distribution of output values from a single simulation may not be normally distributed, the distribution of the mean output values across multiple simulations *is* normally distributed.

Simulated market returns are based on annual data from Ibbotson (2007), and asset-class return relatives, (1 + return), are log-normally distributed. The arithmetic mean inflation-adjusted returns to large-cap stocks and intermediate-term U.S. government bonds over the years 1926 to 2006 are 9.1% and 2.4%, respectively, and the standard deviations of the inflation-adjusted returns are 20.2% and 6.8%, respectively. The historic correlation between the returns to large-cap stocks and intermediate-term bonds is 0.14, and that correlation is retained in simulated returns. Similarly, the historic one-year serial correlation in intermediate-term bond returns, 0.23, is maintained in the simulated intermediate-term bond returns. The randomly generated asset-class returns exhibit the same distributional properties and correlations as the historic returns, and the Kolmogorov-Smirnov goodness of fit test concludes that the distributions of simulated

---

[2] Latin Hypercube sampling insures that sampling from a probability distribution uniformly spans the range of possible values.

[3] 50,000 iterations stabilizes optimized variables. The coefficients of variation for minimized probabilities of ruin, for example, are of the order of magnitude 0.01.

[4] A reported average probability of ruin, for example, is the average, across 40 simulations, of the probability of ruin from 50,000 iterations within a single simulation.

asset-class returns are *not* different from the distribution of actual historic asset-class returns. The simulated returns, descriptive of historic asset-class returns, are assumed to represent future asset-class returns, as well. The simulated returns are to, and withdrawals are from, tax-deferred accounts. Portfolios are assumed to be rebalanced annually and transactions costs are ignored.

3.1 Constant inflation-adjusted withdrawals until death

The first model assumes that all begin their retirement with an initial withdrawal from the retirement portfolio. The after-withdrawal remainder of the portfolio is invested in a mix of large-cap common stocks and intermediate-term U.S. bonds and earns, and the remaining portfolio earns the inflation-adjusted rate of return appropriated weighted to the portfolio composition. The portfolio rate of return is:

$$R_{t,i} = (Wt_E)E_{t,i} + (1- Wt_E)B_{t,i} \qquad (1)$$

where $R_{t,i}$ is the weighted average portfolio return for simulation number i at time t, Wt_E is the portfolio fraction allocated to equities, $E_{t,i}$ is the inflation-adjusted rate of return to large-cap equities for simulation number i at time t, and $B_{t,i}$ is the inflation-adjusted rate of return to intermediate bonds for simulation number i at time t.

Subsequent withdrawals are always the same inflation-adjusted amount from the portfolio measured in inflation-adjusted dollars. Hence, the value of the portfolio is given by equation 2 below:

$$V_{t,i} = [V_{t-1,i} – (W)\ V_0]\ (1 + R_{t-1,i}) \qquad (2)$$

where V designates the value of the portfolio in inflation-adjusted dollars and W is constant withdrawal fraction.

The model contains two conditional variables, $ALIVE_t$ and RUINt, which are either true or false. After the first year, $ALIVE_t$ takes its value (true or false) depending upon a randomly drawn number from the binomial conditional probability of living to year t, having already lived to year (t-1)[5]. A withdrawal is attempted from the portfolio only if ALIVEt is true. RUINt takes its value to indicate portfolio ruin (true) if $V_{t,i}$ - (W) $V_0 < 0$. With the addition of these conditional variables, the first model is summarized in equations 3 and 4 below:

$$V_{t,i} = [V_{t-1,i} - (W)\ V_0]\ (1 + R_{t-1,i}) \text{ if } ALIVE_t = \text{true and } RUIN_t = \text{false}, \tag{3}$$

otherwise

$$V_{t,i} = V_{t-1,i} \tag{4}$$

The t subscript increments to 101 minus the initial retirement age because the mortality table is truncated at age 101, causing $ALIVE_{101,i}$ to always be false. The i subscript increments from 1 to the 50,000 iterations (retirees or portfolios) of the simulation.

A simulation of 50,000 iterations generates a single probability of ruin (the number of ruined portfolios/50,000) which is the outcome from the portfolio allocation, the withdrawal rate, the age at retirement, the stochastic inflation-adjusted portfolio return, and the stochastic occurrence of death. Across a number of simulations, the probability of ruin is a stochastic variable, possessing the common statistics which describe its distribution. Monte Carlo stochastic optimization is employed to identify the portfolio

---

[5] Probabilities are from the mortality table for the total population from the Centers for Disease Control, National Center for Health Statistics for the total U.S. population (2002).

allocation to equities (Wt_E) which minimizes the mean probability of ruin from 40 simulations of 50,000 iterations each. Monte Carlo stochastic optimization is employed in the model to identify the portfolio allocation to equities (Wt_E) which minimizes the mean probability of ruin from 40 simulations of 50,000 iterations. Simulated results of portfolio performance for optimal portfolios are presented in Table 1.

Table 1 inserts here.

Table 1 clearly reveals that greater withdrawal rates require more aggressive portfolio allocations, and the more aggressive allocations subject the portfolio to greater probabilities of failure, even as ending portfolio values are increasing. The table also shows the risk of earlier retirement. Earlier retirement requires more aggressive portfolios to provide withdrawals to death, and the more aggressive portfolios combine with longer and more uncertain life spans to increase the probability of portfolio ruin. Several non-optimal portfolio allocation results are shown at the bottom of Table 1. Portfolios which are under (over) weighted to equities generate lower (higher) ending portfolio values. Non-optimal allocations also increase the probability of ruin, with greater increases under greater combinations of the risks from excessive withdrawal rates and excessive longevity (earlier retirement ages and higher constant withdrawal rates).

The interrelationships among the variables of the model are clarified by the linear regression results presented in Table 2[6]. The ruin-minimizing optimal portfolio allocation to large-cap equities increases about 15/4 = 3.75% for each quarter-point increase in the

---

[6] Nonlinear regression produced very similar results with a slightly lower coefficient of determination.

withdrawal rate and 1.5% per year of earlier retirement. The minimized probability of ruin from the optimal portfolio increases about 9.38/4 = 2.35% per quarter-point increase in the withdrawal rate and 0.9% per year of earlier retirement.

Table 2 inserts here.

3.2 Variable, managed, inflation-adjusted withdrawals until death

Model 2 maintains the same simulated returns as model 1, but it incorporates withdrawal rate changes "triggered" by decision rules as proposed by Pye (2001). The decision rules are similar to the "guardrails" of Guyton (2006) and patterned after Stout and Mitchell (2006).

The foundation of the decision rules is the present value interest factor for an annuity due for the retiree's expected remaining lifetime, L, at the average inflation-adjusted rate of return, Avg r, over historic overlapping time periods of the expected remaining life and portfolio allocation[7]. This factor, $\text{PVIFADue}_{L,\text{Avg r}}$, has theoretical support for use in personal decision-making, reflects financial history, integrates expected longevity, and provides a locus over time to a desired ending portfolio balance.

The simulation model assumes that the retiree, perhaps under the advice of his financial planner, selects the initial withdrawal rate based upon the risk-return tradeoff between higher projected lifetime probabilities of ruin and higher withdrawal rates. The maximum

---

[7] The expected remaining life is computed from the same mortality table is rounded up to the next whole number of years.

permissible initial withdrawal rate would likely be V_0 / $\text{PVIFADue}_{L,\text{Avg r}}$, using expected returns and expected longevity as though they were known with certainty.

Subsequently, the retiree or his planner computes, year-by-year, the portfolio value, V'_t, necessary to sustain the prior year's withdrawal amount, $(W_{t-1})(V_0)$, over the expected remaining lifetime if the portfolio earns the average inflation-adjusted rate of return, Avg r, over historic overlapping time periods of the expected remaining life, L, and portfolio allocation. The required portfolio amount is

$$V'_t = (W_{t-1})\ (V_0)\ (\text{PVIFADue}_{L,\text{Avg r}}). \tag{5}$$

The sufficiency of the retirement portfolio is monitored on the basis of $\text{PVIFADue}_{L,\text{Avg r}}$, and corrective withdrawal rate changes are initiated at signals of portfolio inadequacy or excess accumulation. Withdrawal rate reductions are managed by a set of decision rules, termed *portfolio preservation rules*, designed to reduce or delay portfolio ruin. Following Pye's suggestion, a second set of decision rules, termed *portfolio prosperity rules*, manage withdrawal rate increases. The portfolio prosperity rules are designed to allow affordable withdrawal rate increases while avoiding overreactions which might threaten future portfolio sustainability.

Portfolio preservation rules.

If portfolio under-performance has caused the value of the retirement portfolio at year t, V_t, to be less than V'_t, the withdrawal rate is reduced to the withdrawal rate which *is* *sustainable* over the remaining lifetime at the average inflation-adjusted rate of return over historic overlapping time periods of the expected remaining lifetime, $W_t = [V_t / PVIFADue_{L,Avg\ r}] / V_{0.}$ Without an absolute minimum withdrawal rate, however, prolonged market under-performance may cause the portfolio-amortizing withdrawal rate to drop below a minimally acceptable level. Given information regarding the choice, the retiree must ultimately determine the maximum standard of living sacrifice he is willing to endure to improve portfolio sustainability. Accordingly, the model assumes that the retiree sets the minimally acceptable withdrawal rate, W_{Min}, as an alternative to portfolio ruin, and the simulation uses this personal minimum withdrawal rate as an absolute withdrawal rate floor. The portfolio preservation rules are summarized in equations 6 through 8, below:

Preservation 1: If $V_t < V'_t$, then $W_t = [V_t / PVIFADue_{L,Avg\ r}] / V_0$, (6)

Preservation 2: If $W_t < W_{Min}$, $W_t = W_{Min}$, otherwise (7)

Preservation 3: $W_t = W_{t-1}$ (8)

Portfolio prosperity rules.

In the event that portfolio under-performance signals that the existing withdrawal rate is not sustainable ($V_t < V'_t$), preservation rule 1 forces an immediate and complete recognition of the portfolio deficiency through the withdrawal rate reduction. Similarly, portfolio over-performance may cause the current portfolio value to exceed the value deemed reasonably necessary to sustain the existing withdrawal rate, signaling the potential for a withdrawal rate increase. An immediate and complete increase in the withdrawal rate, however, may threaten the future sustainability of the portfolio as the portfolio lacks any reserve or buffer against future market reversals. To protect against an overreaction that is too quick, the portfolio prosperity rules require that unanticipated increases in the retirement portfolio value remain in the portfolio until an adequate reserve, RES, is accumulated to protect the portfolio from subsequent ruin. The required reserve is a multiple of the required portfolio amount necessary to sustain the portfolio, V'_t. Serving as a pre-condition for any withdrawal rate increase, the reserve requirement limits withdrawal rate increases in response to transitory increases in the portfolio but permits responses to long-term growth trends. For a given initial withdrawal rate, greater portfolio reserves are expected to reduce the probability of ruin or provide greater protection from earlier retirements.

The prosperity rules also protect the survivability of the portfolio from increases in the withdrawal rate which are too complete by permitting only partial adjustment of the withdrawal rate. Upon a signal that portfolio growth has met the reserve pre-condition for an increase, the maximum withdrawal rate that the portfolio would sustain is $(V_t/PVIFADue_{L,Avg\ r}) / V_0$. Complete adjustment to the higher withdrawal rate, however, would cause drastic increases in the withdrawal rate and eliminate the portfolio reserve. In recognition of these problems, the simulation model increases the withdrawal rate only a fraction, PART, towards the maximum rate. Higher adjustment fractions would be associated with higher probabilities of ruin as retirees adjust the withdrawal rate more completely in response to abnormally high market returns or shortened longevity, and therefore, more completely reduce the portfolio buffer against subsequent market declines or unexpected longevity. The size of the portfolio reserve and the rate of partial adjustment interact to protect the sustainability of the portfolio with lower adjustment fractions providing protection for lower portfolio reserves.

The portfolio prosperity rules are presented in equations 9 and 10 below:

Prosperity 1: If $V_t > V'_t\ (1 + \text{RES})$, then

$$W_t = W_{t-1} + (\text{PART})\ \{[(V_t / \text{PVIFADue}_{L,\text{Avg } r}) / V_0] - W_{t-1}\}, \text{ otherwise} \quad (9)$$

Prosperity 2: $W_t = W_{t-1}$ (10)

The minimum acceptable withdrawal rate (W_{Min}) is set at two-thirds of the initial withdrawal rate, and Monte Carlo stochastic optimization is employed in the model to identify the portfolio allocation to equities and the two control parameters, RES and PART, which minimize the mean probability of ruin from 40 simulations of 50,000 iterations. Optimal parameters are presented in Table 3, and simulated results of portfolio performance from the optimal parameters are presented in Table 4.

Table 3 inserts here.

Table 4 inserts here.

A comparison of Tables 1 and 3 exposes the impact of withdrawal management on optimal portfolio allocations. The threat to portfolio ruin results from the interactions of the withdrawal rate, the retirement age, and the portfolio allocation. The greatest risk to age 55 retirees is outliving the portfolio, and optimal management reduces the optimal portfolio allocations to equity (Table 1 to Table 3) at all withdrawal rates presented, with greater reductions at higher withdrawal rates. As the retirement age increases, the longevity risk decreases relative to the risk of excessive withdrawal rates, and the optimal allocations to equity begin to increase as the risk of excess withdrawals surpasses the longevity risk. By age 65, the risk of excessive withdrawal rates dominates the longevity risk and portfolio allocations to equity increase at all withdrawal rates shown, albeit by less at higher initial withdrawal rates.

Table 3 shows that the optimal reserve multiple (RES) increases to protect the sustainability of the portfolio from greater allocations to equities. The optimal rate of adjustment to higher withdrawal rates (PART) seems to be between 0.2 and 0.3, although no correlation to the other variables is obvious. Table 4 shows that greater portfolio protection causes from ruin causes greater ending portfolio values, particularly if the withdrawals threat is compounded by the longevity threat (higher initial withdrawals *and* earlier retirement ages). A comparison of Tables1 and 4, however, shows that the management process has reduced the average probability of ruin by about 50% while reducing the excess accumulation in the ending portfolio by about 50%, and furthermore, the withdrawal management process has generated lifetime average withdrawal rates in excess of the constant withdrawal rates of Table 1 and the initial withdrawal rates of Table 4.

The interrelationships among the variables of the managed withdrawals model are clarified by the linear regression results presented in Table 5. The optimal portfolio allocation to large-cap equities increases about 10.71/4 = 2.7% for each quarter-point increase in the withdrawal rate and 1.4% per year of earlier retirement; and the minimum probability of ruin increases about 4.9/4 = 1.23% per quarter-point increase in the withdrawal rate and 0.4% per year of earlier retirement.

The effects of withdrawal rate management are revealed by comparing the results in Tables 2 and 5. The withdrawal management process (Table 5) has caused the optimal portfolio allocation to equities and the minimum probability of ruin to be less sensitive to the withdrawal rate and the age at retirement. The optimal equities allocation increases only 2.7% per quarter-point increase in the withdrawal rate instead of 4%, and ¼% per year of earlier retirement instead of 1.5%. Concurrently, the minimum probability of ruin increases only 1.23% per quarter-point increase in the withdrawal rate instead of 2.35%, and 0.4% per year of earlier retirement instead of 0.9%.

4. Conclusions

Retirees and perspective retirees face a risk-return tradeoff in selecting higher withdrawal rates from the retirement portfolio, the risk of running out of money during the remaining retirement lifetime versus the return of improved retirement lifestyle. Informed of the alternatives, the decision is ultimately that of the retiree. This research is part of the growing body of literature which contributes to the requisite risk-return information.

A first step in improving the risk-return alternatives is selecting the optimal portfolio allocation to expected retirement lifespan, and Monte Carlo stochastic optimization is appropriate to determining the portfolio allocation which minimizes the probability of portfolio ruin. The increased probability of ruin cost for the improved lifestyle benefit from a higher withdrawal rate is thereby minimized. Both the ruin-minimizing portfolio allocation and the minimized probability of ruin are decreasing functions of the retirement age and increasing function of the withdrawal rate.

A second step in improving the risk-return alternatives is withdrawal management. A set of decision rules to meet the objectives of the retiree is adopted, and Monte Carlo stochastic optimization is used to select the decision rules parameters to minimize the probability of portfolio ruin and the optimal portfolio allocation. Optimal parameter values for an illustrative portfolio performance-based withdrawal rate management scheme generate optimal managed withdrawals from optimal portfolios. The illustrative optimal withdrawal management from optimal portfolios reduces the probability of ruin and the excess portfolio accumulation by about 50%. The ruin-minimizing portfolio allocation and the minimized probability of ruin remain decreasing functions of the retirement age and increasing functions of the initial withdrawal rate. However, optimal management from optimal retirement portfolios reduces the sensitivity of the probability of ruin to higher initial withdrawal rates and earlier retirement ages, and thereby minimizes the cost of the benefit from higher and earlier withdrawals.

Future research directed at the risk-return tradeoff between higher probabilities of ruin and higher or earlier retirement withdrawals should consider Monte Carlo stochastic optimization to minimize the ruin risk from the lifestyle return.

References

Ameriks, J., Veres, R., & Warshawsky, M. J. (2001) Making retirement income last a lifetime. *Journal of Financial Planning*, 14, 60-76.

Bengen, W. P. (1994) Determining withdrawal rates using historical data. *Journal of Financial Planning*, 7, 171-180.

Bengen, W. P. (1996) Asset allocation for a lifetime. *Journal of Financial Planning*, 9, 58-67.

Bengen, W. P. (1997) Conserving client portfolios during retirement, part III. *Journal of Financial Planning*, 10, 84-97.

Bengen, W. P. (2001) Conserving client portfolios during retirement, part IV. *Journal of Financial Planning*, 14, 110-119.

The Centers for Disease Control, National Center for Health Statistics, Total U.S. Population, 2002.

Collins, Patrick J., Lawson, Kristor J. & Chambers, Jon C. (1997) Monitoring retirement portfolio sufficiency. *Journal of Financial Planning*, 10, 66-74.

Cooley, P. L., Hubbard, C. M. & Walz, D. T. (1998) Retirement savings: Choosing a withdrawal rate that is sustainable. *Journal of the American Association of Individual Investors*, 20, 16-21.

Guyton, J. (2004) Decision rules and portfolio management for retirees: Is the ‘safe’ initial withdrawal rate too safe? *Journal of Financial Planning,* 17, 54-62.

Guyton, J. & Klinger, William J. (2006) Decision rules and maximum initial withdrawal rates. *Journal of Financial Planning*, 19, 48-58.

Ibbotson Associates, (2007) Stocks, bonds bills and inflation: The 2001 Yearbook. Chicago: Ibbotson Associates.

Klinger, William J. (2007) Planners demand more from retirement planning software. *Journal of Financial Planning*, 20, 60-67.

McCarthy, Ed (2006) Planners demand more from retirement planning software. *Journal of Financial Planning*, 19, 28-34.

Milevsky, M. E. (2001) Spending your retirement in Monte Carlo. *Journal of Retirement Planning,* 4, 21-29.

Milevsky, M. E. & Robinson, C. (2005) A sustainable spending rate without simulation. *Financial Analysts Journal*, 61, 89-100.

Pye, G. B. (2000) Sustainable investment withdrawals. *Journal of Portfolio Management*, 26, 73-83.

Stout, R. Gene JBD

Stout, R. Gene & Mitchell, John B. (2006) Dynamic retirement withdrawal planning. *Financial Services Review*, 15, 117-131.

Terry, Rory L. (2003) The relation between portfolio composition and sustainable withdrawal rates. *Journal of Financial Planning*, 17, 66-71.

Tezel, A. A. (2004) Sustainable retirement withdrawals. *Journal of Financial Planning*, 17, 52-57.

Table 1

Constant inflation-adjusted withdrawals until death: Optimal portfolio allocations to large-cap stocks and simulation results of the distributions of the probability of ruin and ending portfolio value. Several non-optimal allocations are shown for comparison.

Retire Age	**Infl-adjusted Withdrawal Rate (%)**	**Optimal Wt$_E$ (%)**	**Average Prob of Ruin (%)**	**Std Dev Avg Prob Ruin (%)**	**Average End Portfolio (multiple)**	**Std Dev Avg End Port (multiple)**
55	4.25	66.81	8.56	0.15	3.02	0.023
	4.50	69.73	10.91	0.18	3.04	0.026
	4.75	73.19	13.38	0.15	3.17	0.036
	5.00	76.52	16.01	0.15	3.20	0.030
	5.25	79.00	18.75	0.16	3.21	0.033
	5.50	82.56	21.45	0.24	3.32	0.034
	5.75	86.55	24.13	0.17	3.49	0.055
	6.00	89.59	26.97	0.26	3.57	0.065
	6.25	91.91	29.67	0.20	3.58	0.064
	6.50	94.58	32.37	0.26	3.67	0.074
60	4.25	55.38	5.68	0.20	1.84	0.013
	4.50	59.80	7.52	0.11	2.00	0.010
	4.75	64.18	9.64	0.10	2.00	0.009
	5.00	68.95	11.92	0.10	2.10	0.013
	5.25	75.00	14.37	0.09	2.22	0.013
	5.50	79.41	16.75	0.13	2.31	0.020
	5.75	83.45	19.30	0.20	2.37	0.026
	6.00	87.00	21.90	0.20	2.54	0.036
	6.25	91.78	24.32	0.20	2.76	0.044
	6.50	95.29	26.72	0.26	2.76	0.039
65	4.25	50.25	3.25	0.15	1.43	0.011
	4.50	53.82	4.55	0.15	1.44	0.007
	4.75	58.62	6.09	0.16	1.43	0.012
	5.00	61.10	7.78	0.12	1.44	0.009
	5.25	64.58	9.73	0.11	1.45	0.011
	5.50	67.16	11.88	0.16	1.43	0.015
	5.75	70.51	13.95	0.17	1.44	0.013
	6.00	74.78	16.20	0.18	1.47	0.017
	6.25	79.81	18.58	0.22	1.53	0.017
	6.50	83.20	20.77	0.17	1.54	0.013

Retire Age	**Infl-adjusted Withdrawal Rate (%)**	**Non-optimal Wt$_E$ (%)**	**Average Prob of Ruin (%)**	**Std Dev Avg Prob Ruin (%)**	**Average End Portfolio (multiple)**	**Std Dev Avg End Port (multiple)**
55	5.00	65.00	16.41	0.22	2.28	0.022
55	5.00	85.00	16.11	0.15	4.07	0.045
55	6.00	79.00	27.85	0.14	2.55	0.035
55	6.00	99.00	27.69	0.22	4.73	0.088
65	5.00	50.00	8.04	0.14	1.15	0.006
65	5.00	70.00	7.89	0.12	4.72	0.015

Table 2
Constant inflation-adjusted withdrawals until death: Linear regression results based upon Table 1 data.

Independent Variable	**Independent Variable Coefficients (Std. error in parentheses)**			**Statistics**	
	Constant	**Retire Age**	**Infl-adjusted Withdrawal Rate (%)**	**R^2**	**F**
Optimal Wt_E (%)	0.818	-0.015	15.00	0.973	480.0***
	(0.066)	(0.001)***	(0.554)***		
Average Prob of Ruin (%)	0.190	-0.009	9.38	0.987	1035.5***
	(0.028)	(0.0004)***	(0.234)***		

*** indicates statistical significance at the .001 confidence level, standard errors in parentheses

Table 3

Variable, managed, inflation-adjusted withdrawals until death: Optimal (ruin minimizing) portfolio allocations, reserve multiples, and partial adjustment fractions if the minimum withdrawal rate is 2/3 of the initial withdrawal rate.

Retire Age	**Initial Infl-adjusted Withdrawal Rate (%)**	**Optimal Wt_E (%)**	**Optimal Portfolio Reserve (RES multiple)**	**Optimal Withdrawal Adjustment (PART fraction)**
55	4.25	61.83	2.132	0.203
	4.50	63.97	2.176	0.262
	4.75	65.98	2.174	0.263
	5.00	68.28	2.213	0.259
	5.25	70.35	2.352	0.218
	5.50	72.34	2.368	0.278
	5.75	74.54	2.405	0.277
	6.00	77.23	2.519	0.249
	6.25	79.84	2.473	0.295
	6.50	82.52	2.529	0.264
60	4.25	56.75	2.251	0.252
	4.50	60.25	2.481	0.298
	4.75	62.58	2.507	0.259
	5.00	65.38	2.734	0.312
	5.25	68.23	2.800	0.305
	5.50	71.54	2.867	0.248
	5.75	73.35	2.905	0.283
	6.00	76.58	3.045	0.292
	6.25	78.17	3.086	0.288
	6.50	81.01	3.185	0.254
65	4.25	56.69	1.725	0.253
	4.50	61.37	1.676	0.243
	4.75	61.38	1.670	0.300
	5.00	62.77	1.549	0.307
	5.25	65.29	1.540	0.307
	5.50	68.45	1.525	0.225
	5.75	73.24	1.508	0.238
	6.00	77.32	1.494	0.242
	6.25	80.71	1.468	0.245
	6.50	85.62	1.453	0.224

Table 4

Variable, managed, inflation-adjusted withdrawals until death: Simulation results of the distributions of the probability of ruin, ending portfolio value, and the lifetime average withdrawal rate from the optimal simulation parameters in Table 3 (the minimum withdrawal rate is 2/3 of the initial withdrawal rate).

Retire Age	**Initial Withdrawal Rate (%)**	**Average Prob of Ruin (%)**	**Std Dev Avg Prob Ruin (%)**	**Average End Portfolio (multiple)**	**Std Dev Avg End Port (multiple)**	**Avg Lifetime withdrawal Rate (%)**	**Std Dev Avg Lifetime withdrawal Rate (%)**
55	4.25	4.09	0.09	1.43	0.010	6.13	0.017
	4.50	5.24	0.12	1.37	0.008	6.35	0.017
	4.75	6.52	0.10	1.36	0.087	6.49	0.015
	5.00	7.79	0.11	1.38	0.012	6.63	0.021
	5.25	9.11	0.11	1.47	0.010	6.65	0.012
	5.50	10.60	0.08	1.40	0.012	6.86	0.017
	5.75	12.15	0.01	1.43	0.011	7.00	0.015
	6.00	13.66	0.13	1.52	0.018	7.10	0.021
	6.25	15.37	0.13	1.48	0.018	7.36	0.015
	6.50	16.86	0.19	1.57	0.012	7.46	0.014
60	4.25	2.94	0.08	1.15	0.006	5.73	0.015
	4.50	3.75	0.13	1.17	0.008	5.94	0.019
	4.75	4.76	0.16	1.20	0.008	6.04	0.016
	5.00	5.88	0.13	1.19	0.008	6.25	0.017
	5.25	7.08	0.13	1.21	0.007	6.41	0.014
	5.50	8.33	0.14	1.28	0.008	6.51	0.010
	5.75	9.64	0.18	1.25	0.007	6.69	0.013
	6.00	9.71	0.15	1.34	0.012	6.81	0.019
	6.25	12.45	0.18	1.29	0.017	6.97	0.018
	6.50	13.92	0.24	1.34	0.020	7.08	0.020
65	4.25	2.14	0.09	0.95	0.005	5.89	0.021
	4.50	2.85	0.10	0.96	0.004	6.16	0.013
	4.75	3.69	0.11	0.90	0.005	6.29	0.020
	5.00	4.65	0.12	0.86	0.005	6.48	0.014
	5.25	5.67	0.14	0.85	0.006	6.62	0.022
	5.50	6.50	0.18	0.89	0.006	6.75	0.022
	5.75	7.72	0.20	0.90	0.007	7.03	0.019
	6.00	8.90	0.24	0.91	0.007	7.28	0.023
	6.25	10.23	0.24	0.91	0.006	7.50	0.023
	6.50	11.66	0.16	0.95	0.010	7.74	0.028

Table 5
Variable, managed, inflation-adjusted withdrawals until death: Linear regression results based upon Table 4 data.

Independent Variable	**Independent Variable Coefficients (Std. error in parentheses)**			**Statistics**	
	Constant	**Retire Age**	**Infl-adjusted Withdrawal Rate (%)**	**R^2**	**F**
Optimal Wt_E (%)	0.270	-0.002	10.71	0.962	341.3^{***}
	(0.049)	$(0.0007)^{***}$	0.413^{***}		
Average Prob of Ruin (%)	0.042	-0.004	4.90	0.976	548.0^{***}
	(0.019)	$(0.0003)^{***}$	0.161^{***}		

*** indicates statistical significance at the .001 confidence level, standard errors in parentheses